MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

EEM1016 – ENGINEERING MATHEMATICS I

(All sections / Groups)

05 OCTOBER 2015
2.30 p.m - 4.30 p.m
(2 Hours)

INSTRUCTION TO STUDENT

1.This Question paper consists of 6 pages (including cover page) with 5 Questions only.

2. Attempt ALL questions. The distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided.

Question 1

(a) Find the first derivative of the following functions:

(i) $f(x) = \dfrac{\sin 2x}{x}$ [3 marks]

(ii) $y(x) = \ln\left(\dfrac{1}{\sqrt{2x^2+1}}\right)$ [4 marks]

(b) Let $f(x) = x^3 - 6x^2 + 11x - 6$ where $x \in [0,3]$

(i) Identify the critical points of $f(x)$ [2 marks]

(ii) Hence, find the local and absolute extreme values of the function. [4 marks]

(c) Evaluate the following integrals:

(i) $\int \dfrac{x+1}{x^2+2}dx$ [4 marks]

(ii) $\int_0^{\pi/2} \cos^3 x\, dx$ [3 marks]

Continued...

Question 2

(a) Find the limit of the following sequence and determine whether it is convergent or divergent.

$$a_n = \lim_{n\to+\infty} \frac{5n^4 + 3n^2}{n^4 + 5n + 7}$$

[3 marks]

(b) A tennis ball is dropped from a height of 60 centimeters. If it rebounds approximately two-thirds the distance after each fall, use an infinite geometric series to approximate the total distance of the tennis ball travels.

[5 marks]

(c) Determine the radius and interval of convergence in the following power series.

$$\sum_{n=0}^{\infty} \frac{n^{10}}{(2n+1)!} x^n$$

[5 marks]

(d) Obtain the Maclaurin series of the function $f(x) = \cos 2x$.

[7 marks]

Continued...

Question 3

(a) Find the limit, if exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{y\sin x}{x^2+3y^2}$$

[5 marks]

(b) The dimension of a closed rectangular box is measured as $x = 60$ cm, $y = 50$ cm and $z = 40$ cm, respectively, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of this box.

[4 marks]

(c) Use implicit differentiation to find $\frac{dy}{dx}$ for the following equation.

$$y\ln(x^2+y^2+1)=8$$

[4 marks]

(d) Use Lagrange multipliers to find the minimum of the following function subject to the given constraint.

$$f(x,y,z)=(x-2)^2+(y-1)^2+(z+1)^2; \qquad x+y-z=1.$$

[7 marks]

Continued...

Question 4

(a) Use de Moivre's Theorem to show that

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta.$$

[4 marks]

(b) Use Euler's Identity to show that

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i},$$
$$\cos x = \frac{e^{ix} + e^{-ix}}{2}.$$

[4 marks]

(c) Express $\ln\left(-\frac{1}{2} - j\frac{1}{2}\sqrt{3}\right)$ in the form of $x + jy$.

[4 marks]

(d) Let ΔABC be the triangle with vertices $A(1,1,8)$, $B(4,-3,-4)$ and $C(-3,1,5)$. Find the angle formed at A.

[4 marks]

(e) Find the parametric form of a line passing through $(-1,2,3)$ that is parallel to the line of intersection of the planes $3x - 2y + z = 4$ and $x + 2y + 3z = 5$.

[4 marks]

Continued...

Question 5

(a) State, without proof, the general formula of a Fourier series on $(-\pi, \pi)$ for a function $f(x)$, and the expressions of its coefficients.

[4 marks]

(b) Find Fourier series of the following function

$$f(x) = \begin{cases} 0 & \text{if } -\pi < x \le 0 \\ -2\pi x & \text{if } 0 < x < \pi \end{cases}$$

where $f(x + 2\pi) = f(x)$.

[13 marks]

(c) Using part (b), or otherwise, show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots.$$

[3 marks]

End of Paper